Math 2263 Section 10 Quiz 8

Name: _____

Time limit: 15 minutes

1. (4 points) The joint density function for a pair of random variables X and Y is

$$f(x,y) = \begin{cases} 4xy & \text{if } 0 \le x \le 1,\ 0 \le y \le 1\,, \\ 0 & \text{otherwise.} \end{cases}$$

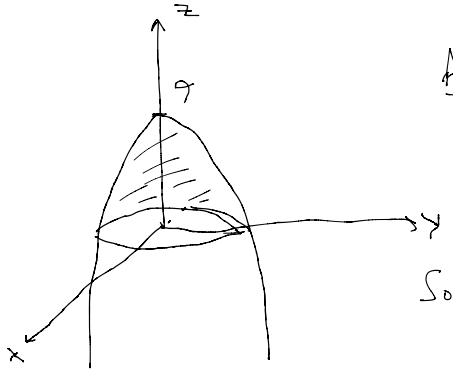
Find the expected value of X.

$$\iint_{\mathbb{R}^2} x f(x,y)\,dA = \int_0^1 \int_0^1 4x^2 y\,dx\,dy$$

$$= \int_0^1 4y \cdot \frac{x^3}{3}\Big|_0^1 dy$$

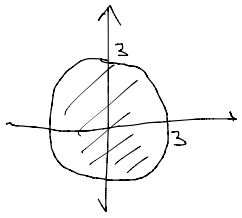
$$= \frac{4}{3}\int_0^1 y\,dy = \frac{4}{3}\cdot\frac{1}{2} = \frac{2}{3}.$$

2. (5 points) Write down an integral **in polar coordinates** that yields the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy-plane. **Do not evaluate the integral.**



Ans: $I = \iint_D \sqrt{1 + f_x^2 + f_y^2}\,dA$ where $f(x,y) = 9 - x^2 - y^2$

D:



So $I = \iint_D \sqrt{1 + \underbrace{(-2x)^2 + (-2y)^2}_{4x^2+4y^2}}\,dA$

polar $\rightarrow$ $= \int_0^{2\pi}\int_0^3 \sqrt{1 + 4r^2}\; r\,dr\,d\theta$

SEE OTHER SIDE FOR MORE PROBLEMS

3. (6 points) Evaluate the iterated integral

$$\int_0^2 \int_0^{2z} \int_0^{\ln x} xe^{-y} dy dx dz .$$

$$= \int_0^2 \int_0^{2z} x \left(\int_0^{\ln x} e^{-y} \, dy \right) dx \, dz = \int_0^2 \int_0^{2z} x \left(-e^{-y} \right) \Big|_{y=0}^{y=\ln x} dx \, dz$$

$$= \int_0^2 \int_0^{2z} x \, e^{-y} \Big|_{y=\ln x}^{y=0} dx dz = \int_0^2 \int_0^{2z} x \left(\underbrace{e^{-0}}_{1} - \underbrace{e^{-\ln x}}_{(e^{\ln x})^{-1} = x^{-1}} \right) dx \, dz$$

$$= \int_0^2 \int_0^{2z} x \left(1 - \frac{1}{x} \right) dx \, dz = \int_0^2 \int_0^{2z} (x-1) \, dx \, dz = \int_0^2 \left(\frac{x^2}{2} - x \right) \Big|_0^{2z} dz$$

$$= \int_0^2 \left(\frac{4z^2}{2} - 2z \right) dz$$

$$= \left(2 \frac{z^3}{3} - z^2 \right) \Big|_0^2$$

$$= \frac{16}{3} - 4 = \frac{4}{3} .$$